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## MIXED CONVECTIVE MHD THREE DIMENSIONAL CASSON FLUID FLOW OVER AN EXPONENTIAL STRETCHING SHEET WITH SLIP EFFECTS AND $N^{\text{TH}}$ ORDER CHEMICAL REACTION

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### ABSTRACT

This present study is dedicated to investigate mixed convective MHD three dimensional Casson fluid flow over an exponentially stretching sheet with slip effects and  $n^{\text{th}}$  order chemical reaction. A mathematical model has developed to describe governing equations under the boundary conditions. The similarity transformations have been used to transform the given partial differential equations to a system of ordinary differential equations and then solved numerically by finite difference method. Non-dimensional parameters which are involved in the derived set of equations are discussed and analyzed through graphs to study the flow characteristics.

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### KEYWORDS:

Magnetohydrodynamics  
(MHD)Casson fluid, chemical  
reaction parameter, Stretching  
sheet.

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## 1. INTRODUCTION

Mostly, the study of non-Newtonian fluids plays an important role and have multiple applications in chemical industries such as melting of polymer solutions, generators, coal slurries, drilling mud, grease and hydrocarbon oils. Due to these broad applications, many researchers are interested towards non-Newtonian fluid flows. One of the interesting non-Newtonian fluid is Casson fluid which has an example as Honey, Soup and concentrated juices etc.

Magyari and Keller (1999) was addressed Similarity solutions of steady boundary layer flow over an exponentially stretching continuous surface with heat and mass transfer. Elbashaeshy (2001) examined numerical analysis of MHD heat transfer flow due to an exponential stretching surface. Influence of buoyancy and viscous dissipation effects on mixed convection flow from an exponentially stretching surface and analyzed both aiding and opposing flow situations was presented by Partha *et al.*, (2005). Khan (2006) addressed a mathematical model to investigate the Viscoelastic boundary layer flow from an exponentially stretching sheet and the solutions obtained by using fourth order R-K method with shooting technique. Sanjayanand and Khan (2006) conferred a model to describe the viscoelastic boundary layer flow with heat transfer due to an exponential stretching sheet to provide the result. Numerical analysis of MHD boundary layer flow towards an exponential stretching sheet with existing of thermal radiation effect was analyzed by Bidin and Nazar (2009). Ishak (2011) discussed the numerical analysis of MHD boundary layer flow over an exponential stretching sheet by using Implicit Finite Difference method. Nadeem *et al.* (2012) analyzed the MHD flow of a Casson fluid over an exponentially shrinking sheet. The Adomian Decomposition Method (ADM) has been employed to obtain the analytical solution. Rahman *et al.*, (2014) using Buongiorno's model to examine the problem of MHD boundary layer flow past an exponential Shrinking/stretching surface with considering of Second order slip. Krishna Jyothi *et al.*, (2015) inspected viscous dissipation effects on MHD flow towards an exponential stretching sheet with thermophoresis diffusion. The impact of chemical reaction on MHD Casson fluid flow caused by a stretching sheet with heat and mass transport was examined by Prasanna and Gangadhar (2015). Krishna Jyothi *et al.*, (2015) inspected viscous dissipation effects on MHD flow towards an exponential stretching sheet with thermophoresis diffusion. AurangZaibet *et al.*, (2016) have been reported the dissipation effects on MHD non-Newtonian Casson fluid flow over an exponential permeable shrinking sheet by using

Shooting technique. The Maxwell nanofluid model for MHD boundary layer slip flow over an exponentially stretching surface with thermophoresis and Brownian effects was studied by Reddy *et al.* (2016). Sandeep *et al.* (2016) was examined an Unsteady MHD radiative flow and Heat transfer of a dusty nanofluid over an exponentially stretching sheet. It is indicated that the enhancement in fluid particle interaction increases the heat transfer rate and depreciates the wall friction. Ullah *et al.*, (2016) constructed a Casson fluid model to analyzed the Slip flow over a nonlinearly elastic sheet with chemical reaction and heat generation/absorption using Keller box numerical approach. Nagalakshmi *et al.*, (2017) numerically investigated the non-Newtonian Casson fluid flow due to an exponentially stretching sheet with existing of ohmic effects. Sulochana *et al.*, (2016) used a bvp4c solver MATLAB Package for MHD cross diffusion effects on MHD nanofluid flow due to an exponential stretching surface problem. Hayat *et al.*, (2017) have selected NDSolve technique to test the third grade fluid flow due to an exponential stretching sheet with considering of Magnetic field effect. Patil *et al.*, (2017) reported mixed convection flow from an exponentially stretching sheet using implicit finite difference method with Newton's linearization technique. Radha *et al.*, (2017) concerned thermal conductivity and slip flow effects on MHD Casson fluid flow due to an exponentially stretching surface using MATLAB bvp4c. Punithavalli *et al.* (2018) analyzed Micropolar fluid over an exponentially stretching sheet with chemical reaction. Anuradha *et al.* (2018) investigated Micropolar Stagnation point fluid flow through exponentially stretching surface with binary chemical reaction.

By motivation of these studies and important application towards casson fluid flow, this present research extends the work of Kankanala Sharada *et al.* (2016) to analyze the effect of investigate MHD three dimensional Casson fluid flow over an exponentially stretching sheet with slip effects and  $n^{\text{th}}$  order chemical reaction. The main motivation of this article is to discuss the characteristics of chemical reaction parameters.

## 2. MATHEMATICAL FORMULATION

Mixed Convective MHD three dimensional Casson fluid flows over an exponentially stretching sheet with slip effects and  $n^{\text{th}}$  order chemical reaction has been considered. The steady, three dimensional, laminar, incompressible flows are taken into account. Cartesian coordinate axes  $(x, y, z)$  considered with corresponding velocities  $(u, v, w)$  and the sheet has stretched along  $xy$  plane at  $z = 0$ . The uniform magnetic field  $B_0$  is applied towards

the positive direction of  $z$  axis. Assume that the exponential form of stretching velocities of the sheet are  $U_w = U_0 e^{\frac{x+y}{L}}$  and  $V_w = V_0 e^{\frac{x+y}{L}}$  respectively. The Casson fluid model expressed as follows

$$\tau_{ij} = \begin{cases} 2 \left( \mu_B + \frac{P_z}{\sqrt{2\pi}} \right) e_{ij}, \pi > \pi_c \\ 2 \left( \mu_B + \frac{P_z}{\sqrt{2\pi}} \right) e_{ij}, \pi < \pi_c \end{cases} \quad (1)$$

Where  $\pi = e_{ij}$ ;  $e_{ij} = (i, j)$  th component of the deformation rate;  $\pi =$  The product of the component of the deformation rate,  $\pi_c =$  Critical value of this product based on the non-Newtonian fluid,  $\mu_B =$  Plastic dynamic viscosity,  $P_z =$  Yield stress of the fluid. This Mathematical model has been formulated to express the governing boundary layer equations as follows;

Continuity Equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial w}{\partial z} = 0 \quad (2)$$

Momentum Equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \mathcal{G} \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial z^2} + g \beta_T (T - T_\infty) + g \beta_c (C - C_\infty) - \frac{\sigma B_0^2}{\rho} u \quad (3)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \mathcal{G} \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho} v \quad (4)$$

Energy Equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_m \frac{\partial^2 T}{\partial z^2} + \frac{Q}{\rho c_p} (T - T_\infty) \quad (5)$$

Concentration Equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} - k_1 (C - C_\infty)^n \quad (6)$$

With associated boundary conditions

$$\left. \begin{aligned} u = U_w = U_0 e^{\frac{x+y}{L}}, v = V_w = V_0 e^{\frac{x+y}{L}}, w = 0, \\ T = T_w = T_\infty + T_0 e^{A \left( \frac{x+y}{2L} \right)} + L_1 \frac{\partial T}{\partial z}, C = C_w = C_\infty + C_0 e^{B \left( \frac{x+y}{2L} \right)} + L_2 \frac{\partial C}{\partial z} \end{aligned} \right\} \text{at } z = 0 \quad (7)$$

$$u = 0, \quad v = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } z \rightarrow \infty$$

The following similarity transformations have used to convert the above partial differential equations (2)-(6) into the set of ordinary differential equations by using the similarity variable  $\eta$ , dimensionless stream functions  $f(\eta)$ ,  $g(\eta)$ , dimensionless temperature  $\theta(\eta)$  and dimensionless concentration  $\phi(\eta)$ .

$$\eta = \left( \frac{U_0}{2\nu L} \right)^{1/2} e^{\frac{x+y}{2L}} z, u = U_0 e^{\frac{x+y}{L}} f'(\eta), v = V_0 e^{\frac{x+y}{L}} g'(\eta)$$

$$w = - \left( \frac{\nu U_0}{2L} \right)^{1/2} e^{\frac{x+y}{L}} (f + \eta f' + g + \eta g'), \theta(\eta) = \frac{T - T_\infty}{T_0 e^{\frac{A(x+y)}{2L}}}, \phi(\eta) = \frac{C - C_\infty}{C_0 e^{\frac{B(x+y)}{2L}}} \quad (8)$$

The equations (3)-(6) becomes

$$\left( 1 + \frac{1}{\beta} \right) f''' + 2(f' + g')f'' + (f + g)f''' + 2\lambda\theta + 2\delta\phi - M^2 f' = 0 \quad (9)$$

$$\left( 1 + \frac{1}{\beta} \right) g''' + 2(f' + g')g'' + (f + g)g''' - M^2 g' = 0 \quad (10)$$

$$\theta'' + \text{Pr} \left[ (f + g)\theta' - A(f' + g')\theta + S\theta \right] = 0 \quad (11)$$

$$\phi'' + \text{Sc} \left[ (f + g)\phi' - B(f' + g')\phi - \gamma\phi'' \right] = 0 \quad (12)$$

The transformed boundary conditions are

$$f(0) = 0, g(0) = 0, f'(0) = 1, g'(0) = \alpha, \theta(0) = 1 + \gamma_1 \theta'(0), \phi(0) = 1 + \gamma_2 \phi'(0) \quad \text{at } \eta = 0$$

$$f'(\infty) \rightarrow 0, g'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0 \quad \text{at } \eta \rightarrow \infty \quad (13)$$

Where the non-dimensional parameters  $M$ ,  $\text{Pr}$ ,  $\text{Sc}$ ,  $A$ ,  $B$ ,  $S$ ,  $\beta$ ,  $\gamma_1$ ,  $\gamma_2$  and  $\gamma$  denote the magnetic parameter, Prandtl number, Schmidt number, temperature exponent, concentration exponent, heat source parameter, Casson fluid parameter, temperature slip parameter, concentration slip parameter and chemical reaction parameter respectively.

### 3. METHOD OF SOLUTION

To solve this present mathematical model, the governing system of partial differential equations with associated boundary conditions have been transformed into set of coupled non-linear boundary layer equations (9)-(12) with linked boundary condition (13) by similarity transformation. The transformed system of governing equations is solved numerically by finite difference method. In order to analyze the physical insight of the flow characteristics and mixed convective parameters, the non-dimensional parameters involved in the velocity, temperature and concentration profiles are assessed numerically

and presented in the form of graphs. This study is extension of Madhusudhana Rao (2018) for MHD mixed convective three-dimensional casson fluid flow over an exponentially Stretching sheet with slip effects and nth order chemical reaction.

#### 4. RESULTS AND DISCUSSION

Main objective of this paper is to study the effect of MHD mixed convective three-dimensional casson fluid flow over an exponentially Stretching sheet with slip effects and nth order chemical reaction. For this purpose, the numerical computations are performed for various values of non-dimensional parameters such as Casson parameter  $\beta$ , thermal exponent A, Concentration exponent B, mixed convection parameters  $\lambda$  and  $\delta$ , Magnetic parameter M, Prandtl number Pr, Schmidt number Sc, chemical reaction parameter  $\gamma$ , thermal slip parameter  $\gamma_1$ , concentration slip parameter  $\gamma_2$  and heat source S with the help of graphs.

Figures 1-2 illustrate the behavior of the casson parameter  $\beta$  on the velocities  $f'(\eta)$  and  $g'(\eta)$ . Increasing values of casson parameter  $\beta$  decrease both velocity profiles. It is analyzed that influence of casson parameter  $\beta$  reduces the momentum boundary layer thickness which impact less resistance to fluid motion due to the stress.

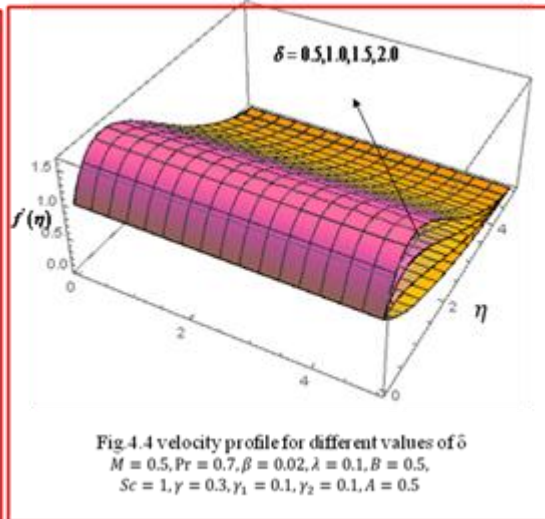
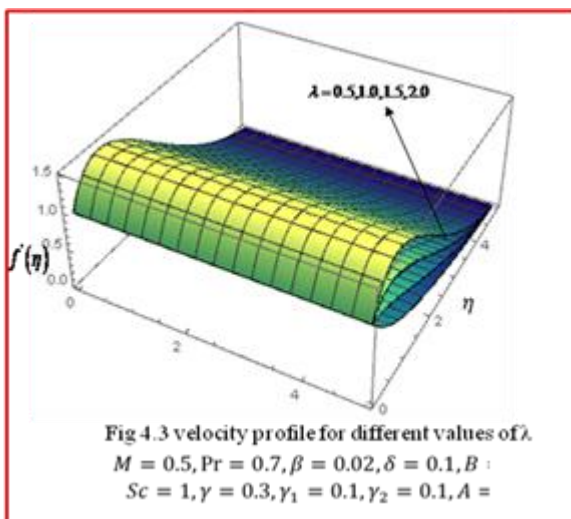
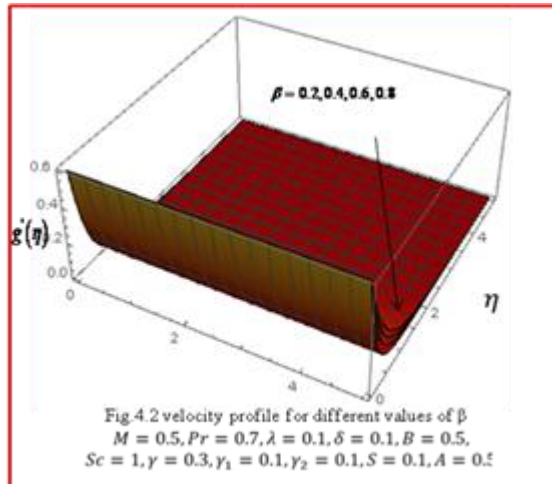
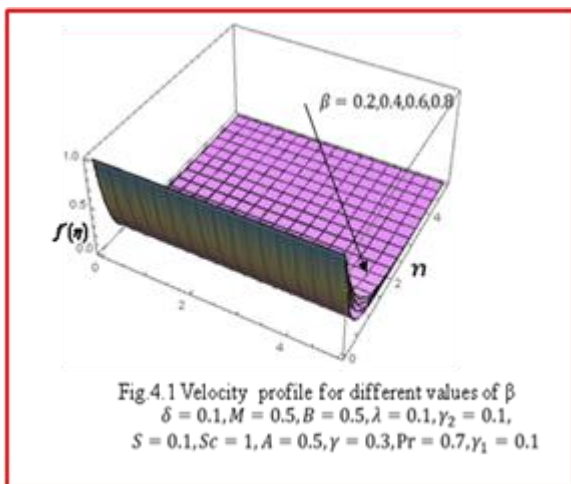
Figure 3- 4 show the effect of mixed convection parameters  $\lambda$  and  $\delta$ , on velocity profile. Increasing values of mixed convection parameters  $\lambda$  and  $\delta$  increase the velocity profile due to the buoyancy effect.

Figure 5-6 depict the influence of magnetic parameter M on the velocities  $f'(\eta)$  and  $g'(\eta)$ . Increasing values of magnetic parameter M decrease the velocity profile due to Lorentz force and decreases the boundary layer thickness.

Figure 7-10 demonstrate the behavior of the non-dimensional variables Prandtl number Pr, thermal exponent A, thermal slip parameter  $\gamma_1$  and heat source S. It is observed that the larger values of Prandtl number Pr, thermal exponent A, thermal slip parameter  $\gamma_1$  reduce the temperature profile and reverse effect in temperature profile for the larger values of heat source S. It has been observed that thermal diffusivity decreases by increasing values of Prandtl number Pr and temperature is increased by heat generation parameter in the fluid.

Figure 11-14 illustrate the influence of Schmidt number Sc, chemical reaction parameter  $\gamma$ , concentration slip parameter  $\gamma_2$  and Concentration exponent B. Increasing values of non-dimensional parameters Schmidt number Sc, chemical reaction parameter  $\gamma$

, concentration slip parameter  $\gamma_2$  and Concentration exponent B decrease the concentration profile. We note that smaller values of concentration exponent and concentration slip parameter gives rise to be stronger the concentration field.





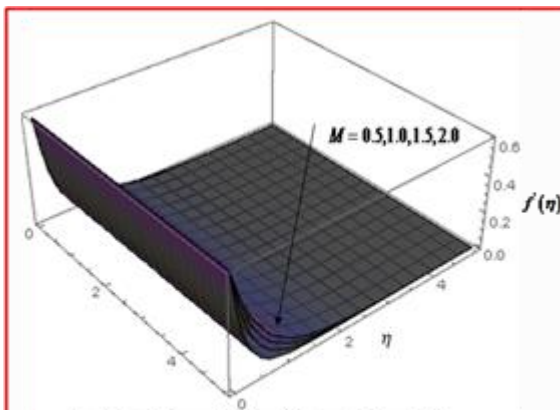


Fig.4.5 velocity profile for different values of  $M$   
 $\delta = 0.1, Pr = 0.7, \beta = 0.02, \lambda = 0.1, B = 0.5,$   
 $Sc = 1, \gamma = 0.3, \gamma_1 = 0.1, \gamma_2 = 0.1, A = 0.5$

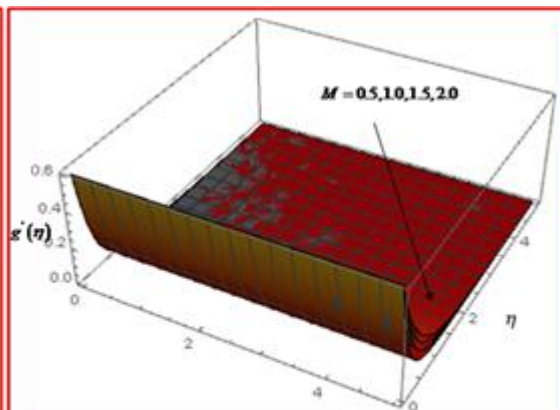


Fig.4.6 velocity profile for different values of  $M$   
 $\delta = 0.1, Pr = 0.7, \beta = 0.02, \lambda = 0.1, B = 0.5,$   
 $Sc = 1, \gamma = 0.3, \gamma_1 = 0.1, \gamma_2 = 0.1, A = 0.5$

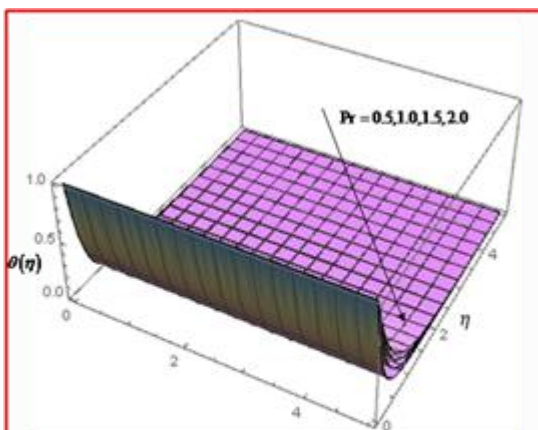


Fig.4.7 Temperature profile for different values of  $Pr$   
 $\delta = 0.1, M = 0.5, \beta = 0.02, \lambda = 0.1, B = 0.5,$   
 $Sc = 1, \gamma = 0.3, \gamma_1 = 0.1, \gamma_2 = 0.1, A = 0.5$

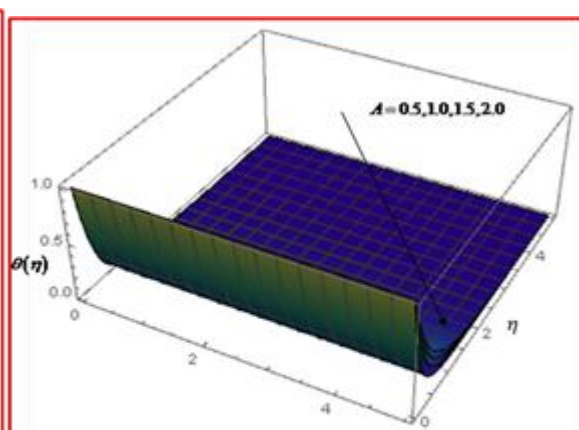


Fig.4.8 Temperature profile for different values of  $A$   
 $\delta = 0.1, M = 0.5, \beta = 0.02, \lambda = 0.1, B = 0.5,$   
 $Sc = 1, \gamma = 0.3, \gamma_1 = 0.1, \gamma_2 = 0.1, Pr = 0.7$

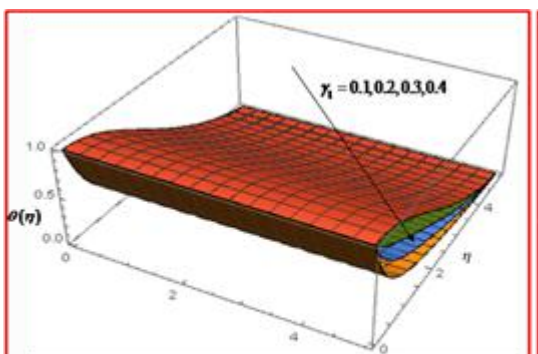


Fig.4.9 Temperature profile for different values of  $\gamma_1$   
 $\delta = 0.1, M = 0.5, \beta = 0.02, \lambda = 0.1, B = 0.5,$   
 $Sc = 1, \gamma = 0.3, A = 0.5, \gamma_2 = 0.1, Pr = 0.7$

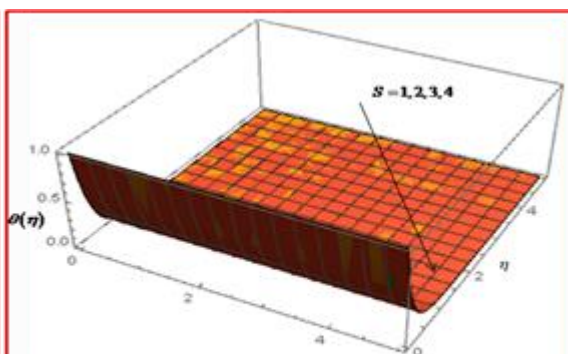
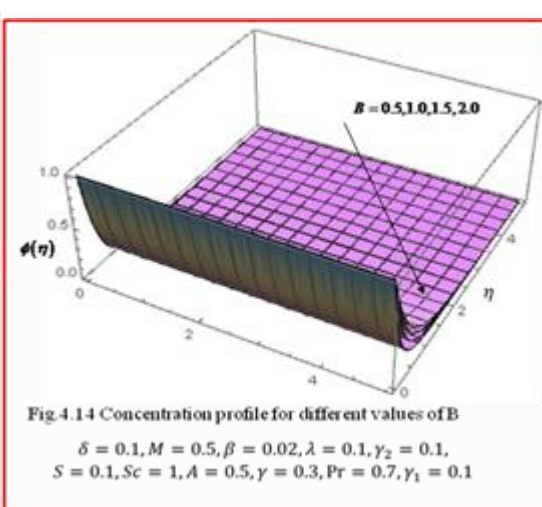
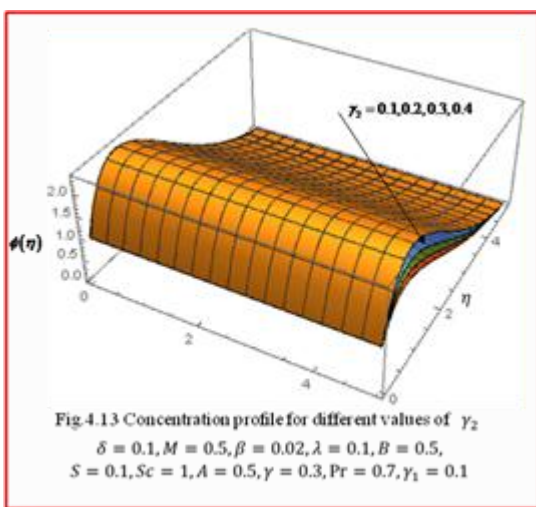
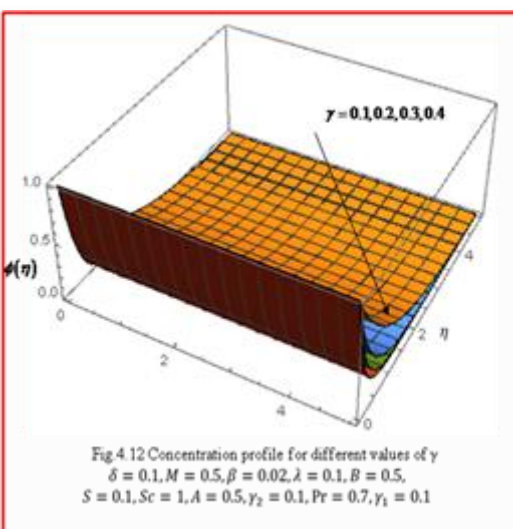
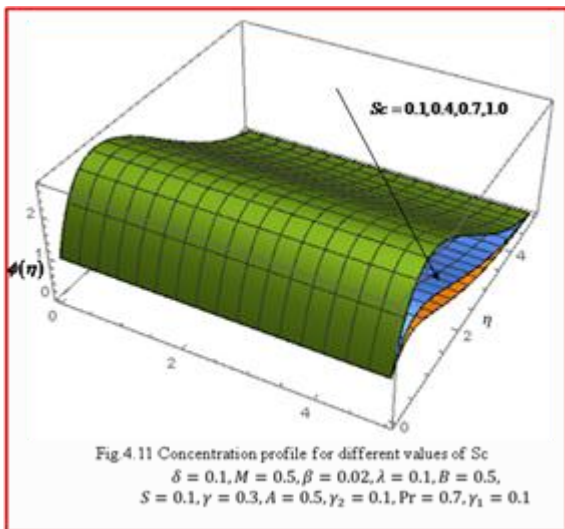


Fig.4.10 Temperature profile for different values of  $S$   
 $\delta = 0.1, M = 0.5, \beta = 0.02, \lambda = 0.1, B = 0.5,$   
 $Sc = 1, \gamma = 0.3, A = 0.5, \gamma_2 = 0.1, Pr = 0.7, \gamma_1 = 0.1$





### 5. CONCLUSION

MHD mixed convective three-dimensional casson fluid flow over an exponentially Stretching sheet with slip effects and nth order chemical reaction has studied in this paper. The conclusions are as follows:

- Increasing values of casson parameter  $\beta$  decrease both velocity profiles. It is analyzed that influence of casson parameter  $\beta$  reduces the momentum boundary layer thickness which impact less resistance to fluid motion due to the stress.
- Increasing values of mixed convection parameters  $\lambda$  and  $\delta$  increase the velocity profile due to the buoyancy effect.
- Increasing values of magnetic parameter  $M$  decrease the velocity profile due to Lorentz force and decreases the boundary layer thickness.
- Larger values of Prandtl number  $Pr$ , thermal exponent  $A$ , thermal slip parameter  $\gamma_1$  reduce the temperature profile and reverse effect in temperature profile for

the larger values of heat source  $S$ . It has been observed that thermal diffusivity decreases by increasing values of Prandtl number  $Pr$  and temperature is increased by heat generation parameter in the fluid.

- Increasing values of non-dimensional parameters Schmidt number  $Sc$ , chemical reaction parameter  $\gamma$ , concentration slip parameter  $\gamma_2$  and Concentration exponent  $B$  decrease the concentration profile. We note that smaller values of concentration exponent and concentration slip parameter gives rise to be stronger the concentration field.

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